Amendments to the Specifications:

Please delete the present title and substitute the following therefore:

**IMAGING OF A SCATTERING MEDIUM USING
THE INTEGRO DIFFERENTIAL EQUATION OF RADIATIVE TRANSFER**

Please replace the paragraph beginning at line 18 of page 3 with the following amended paragraph:

These known techniques, however, have several disadvantages. First, all of the known reconstruction schemes are generally based on the diffusion approximation to the integro-differential equation of radiative transfer. The integro-differential equation of radiative transport describes the propagation of photons through a scattering medium based in part on the internal properties of the target. The diffusion approximation to this equation, makes several assumptions that reduce the complexity of the equation making it easier and faster to solve. However, the diffusion approximation is only valid for example: (1) for highly scattering media where the properties of the medium μ'_s (the scattering coefficient) is much larger than μ_a (the absorption coefficient), (2) for media that do not contain strong discontinuities in optical properties, such as very low scattering and absorbing regions ("void-like regions") embedded in highly scattering materials, and (3) for large source-detector pair separation, i.e., the separation between a source location and a detector location on the target medium.

Please replace the paragraph beginning at line 11 of page 5 with the following amended paragraph:

The present method and system satisfies these needs by providing a gradient-based iterative reconstruction algorithm for imaging of scattering media using (1) the integro-differential equation of radiative transfer as a forward model, and (2) an adjoint differentiation method for the fast and efficient gradient calculation used in the modification of the initial guess in the updating scheme.

Please replace the paragraph beginning at line 16 of page 5 with the following amended paragraph:

One embodiment of the present method and system provides a method for making an initial guess of the internal properties of a target medium, predicting the propagation of energy through the medium based on the initial guess, measuring the actual propagation of energy through the medium, updating the initial guess based on the predicted data and measured data. The predicted propagation of energy is calculated using the integro-differential equation of radiative transfer and an initial guess as to what the spatial properties of the target might be. The measured propagation is obtained by directing energy into the target and measuring the energy emerging from the target. An objective function is then generated that relates the predicted values to the measured values. The objective function gives an indication of how far the initial guess of the spatial properties of the target was from the actual spatial properties of the target. A gradient of the objective function is then generated using a method of adjoint differentiation. The gradient indicates how that initial guess should be adjusted to make the predicted data more closely match the measured data. This process is then repeated until the predicted and measured data are within an acceptable error. At this point the

adjusted initial guess is representative of the spatial properties of the actual target and an image is generated therefrom.

Please replace the paragraph beginning at line 10 of page 7 with the following amended paragraph:

The present method and system employs an iterative image reconstruction scheme that has three major elements. The following sections describe in detail how the three major elements of the present method and system work together to yield a reconstructed image of the internal properties of the scattering target medium. First, the solution of the forward model of the integro-differential equation of radiative transfer is solved using an upwind difference scheme, even-parity finite-element formulation or the like; by way of example, the present invention is illustrated using the upwind scheme. This is followed with a detailed description of the second and third component, namely the objective function and the updating scheme, in which adjoint differentiation is used for the gradient calculation. Referring to FIG. 1, these elements are illustrated as the forward model 105, the objective function/analysis scheme 120 and the updating scheme 125.

Please replace the paragraph beginning at line 21 of page 7 with the following amended paragraph:

The forward model starts at step 100 with an initial guess $\mu_0 = [\mu_{s.0}(r)$, $\mu_{a.0}(r)]$ (scattering and absorption coefficients respectively) of the internal properties, known energy source $S(r_s)$ at the positions r_s and given boundary conditions. Based on

the initial guess μ_0, energy source $S(r_s)$ and the given boundary conditions the forward

model (1) gives a numerical solution of the energy distribution $\Psi(r)$ inside the target

scattering medium, and (2) predicts the energy radiance Ψ_d on the boundary Ω of the

medium based on the <u>integro-differential</u> equation of radiative transfer. Referring to step

110 of FIG. 1, the vector **P**, generated by the forward model, contains the predicted

radiance value Ψ_d, or derived parameter related thereto, at each detector, for each source

detector pair, as a function of the properties μ. Measured data for the actual target to be

imaged is then collected in step 115. The measured radiance at each detector, for each

source detector pair is stored in the vector **M**. A typical imaging system for collecting the

measured data is presented in detail below.

Please replace the paragraph beginning at line 21 of page 10 with the

following amended paragraph:

In the present method and system, the <u>integro-differential</u> equation of

radiative transfer is used for the forward model. Referring to FIG. 1, steps 100 through

110, the radiative transfer equation is used to predict the detector readings of energy

emerging at one or more detector locations on the scattering medium based on an initial

guess of the properties of the medium. The <u>integro-differential</u> equation of radiative

transfer is derived by considering energy conservation in a small volume. The equation

for time-dependent case can be written as:

$$\frac{1}{c}\frac{\partial \Psi(\mathbf{r},\omega,t)}{\partial t} = S(\mathbf{r},\omega,t) - \omega \cdot \nabla \Psi(\mathbf{r},\omega,t) - (\mu_a + \mu_s)\Psi(\mathbf{r},\omega,t) + \mu_s \int_0^{2\pi} p(\omega,\omega')\Psi(\mathbf{r},\omega',t)d\omega'. \quad (1a)$$

and; for the time-independent case can be written as:

$$\omega\nabla\Psi(\mathbf{r},\omega) + (\mu_a + \mu_s)\Psi(\mathbf{r},\omega) = S(\mathbf{r},\omega) + \mu_s \int_0^{2\pi} p(\omega,\omega')\Psi(\mathbf{r},\omega')d\omega', \quad (1b)$$

where **r** is the spatial position vector, ω is a unit vector pointing in the

direction of photon propagation, $\Psi(\mathbf{r},\omega,t)$ and $\Psi(\mathbf{r},\omega)$ are the energy radiance in units of

$W\ cm^{-2}\ sr^{-1}$. $S(\mathbf{r},\omega,t)$ and $S(\mathbf{r},\omega)$ are the source terms representing energy directed into a

solid angle centered on ω in a unit volume at **r**, μ_s is the scattering coefficient given in

units of cm^{-1}, μ_a is the absorption coefficient given in units of cm^{-1} and $p(\omega,\omega')$ is the

phase function that describes the probability that a photon with direction ω' will be

scattered into the direction ω during a scattering event.

Please insert the following paragraph immediately before the paragraph

beginning at line 14 of page 11:

The second term ($\omega\ \nabla\Psi(r,\omega,t)$) on the right hand side of equation (1a) is

commonly referred to as the "streaming term," since it represents the rate of change of

the photon density at spatial position **r** resulting from "streaming" of the photons with

speed **v**, that is, motion in a straight line without collision.(E.g. Transport Calculations for

Nuclear Analyses: theory and Guidelines for Effective Use of Transport Codes, Los

Alamos National Laboratory Report LA-10983-MS, UC-32, Issued: September 1987)

This term contains the differential operator ∇, which generated the spatial derivative of

the energy radiance Ψ with respect to **r**. The last term on the right hand side of equation

(1a) is an integral term that accounts for all photons coming from a direction ω' and are

scattered into the direction ω, therefore adding to the energy radiance Ψ in the direction

ω.

Please replace the paragraph beginning at line 10 of page 12 with the following amended paragraph:

Various computational techniques exist, such as those disclosed by Lewis E E, Miller W F, Computational Methods of neutron transport, New York, John Wiley & Sons, 1984, that numerically solve the integro-differential equation of radiative transfer. Techniques commonly applied include the singular eigenfunction method, the method of spherical harmonics, the method of characteristics, the finite-element method, and the finite-difference discrete-ordinate method. A concise review of these techniques has been presented by Sanchez R, McCormick N J, A Review of neutron transport approximations. Nucl. Sci. Eng. 1982,80:481-535; and McCormick NJ, Inverse radiative transfer problems: a review, Nucl. Sci. Eng. 1992,112:185-198. The discrete-ordinates method is widely used with several different finite-difference approximations, such as the diamond-difference scheme, the weighted diamond-difference scheme, or the centered-difference scheme.

Please replace the paragraph beginning at line 22 of page 12 with the following amended paragraph:

Another means of solving the integro-differential equation of radiative transfer is the upwind-difference scheme used in connection with the discrete-ordinates method to the integro-differential equation of radiative transfer disclosed by Sewell G., The numerical solution of ordinary and partial differential equations, San Diego, Academic Press, 1988. This method is a computationally efficient method for the calculation of the radiance and has the advantage that it easily supplies the required

mathematical structure for the adjoint differentiation calculation. The adjoint

differentiation method is crucial for obtaining the gradient of the objective function in an

computationally efficient way, and thus obtaining an update of the initial guess of the

properties of the target medium. The adjoint differentiation method and the objective

function will be described in detail below in Section IV.

Please replace the paragraph beginning at line 10 of page 13 with the

following amended paragraph:

Returning to the forward problem, to solve the <u>integro-differential</u>

equation of radiative transfer with an upwind-difference discrete-ordinates method, the

angular and spatial variables have to be discretized. First, the integral term in equation

1b is replaced by a quadrature formula that uses a finite set of K angular directions ω_k

with $k = 1, ... K$. This yields a set of K coupled ordinary differential equations for the

angular-dependent radiance $\Psi_k(\mathbf{r}) = \Psi(\mathbf{r}, \omega_k)$ in the directions ω_k. The coupling term is

the internal source term $\mu_s \sum\limits_{k'=1}^{K} a_{k'} p\left(\omega_{k'}, \omega_k\right) \Psi(\mathbf{r}, \omega')$. The parameter $a_{k'}$ is a weighting

factor that depends on the chosen quadrature formula. In this work the extended

trapezoidal rule is employed.